ME202: ADVANCED MECHANICS OF SOLIDS

MODULE – I

Introduction to Stress Analysis & Displacement Field

ME010 306(CE) STRENGTH OF MATERIALS & STRUCTURAL ENGINEERING

Course Objectives:

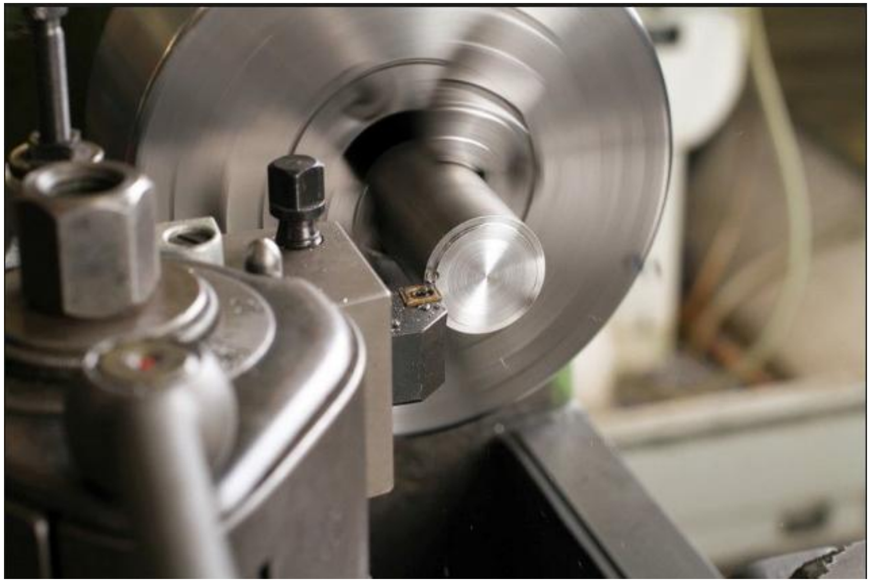
1. To impart concepts of stress and strain analyses in a solid.
2. To study the methodologies in theory of elasticity at a basic level.
3. To acquaint with the solution of advanced bending problems.
4. To get familiar with energy methods for solving structural mechanics problems.

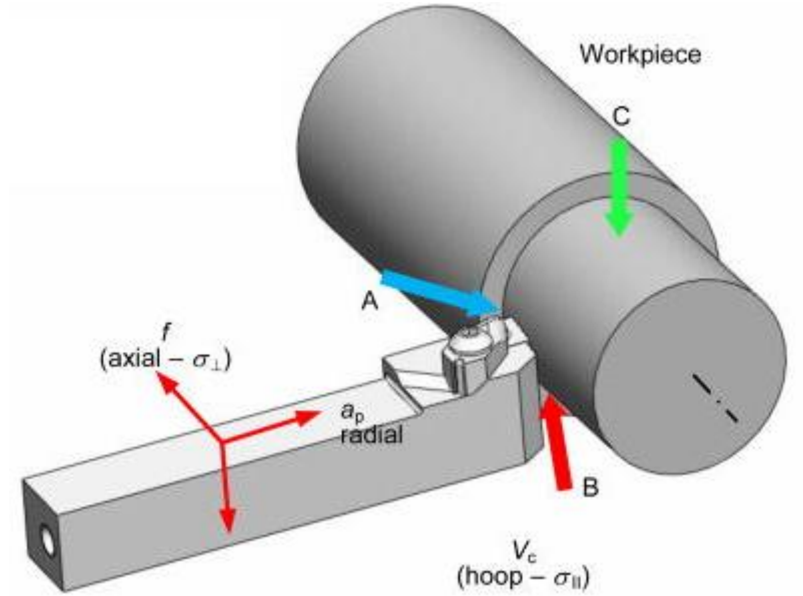
Module – I

1. Introduction to stress analysis in elastic solids
2. Stress at a point - Stress tensor
3. Stress components in rectangular and polar coordinate systems
4. Cauchy's equations
5. Stress transformation.
6. Principal stresses and planes.
7. Hydrostatic and Deviatoric stress components, Octahedral shear stress
8. Equations of equilibrium.

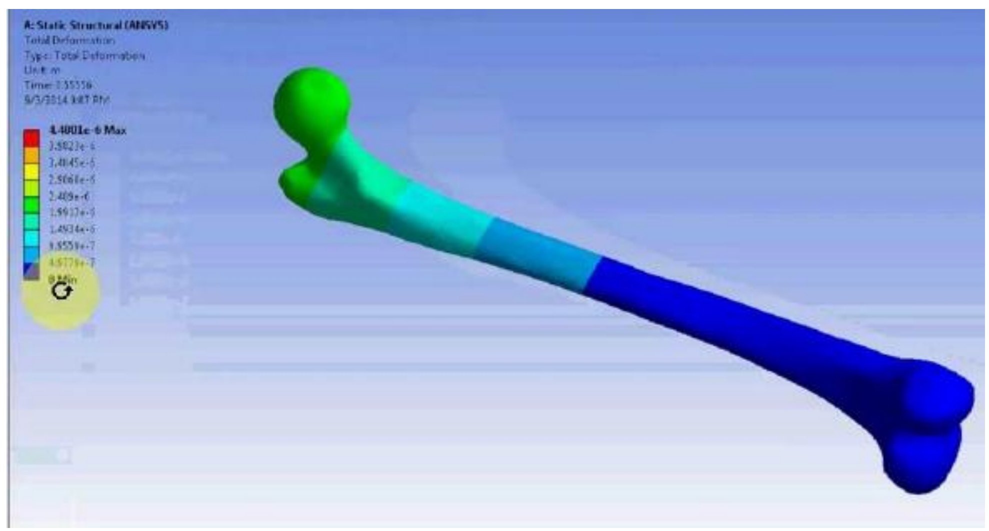
STRUCTURAL FAILURES

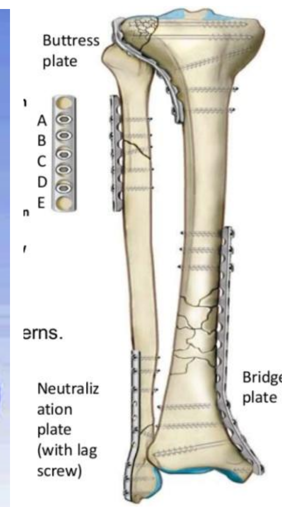


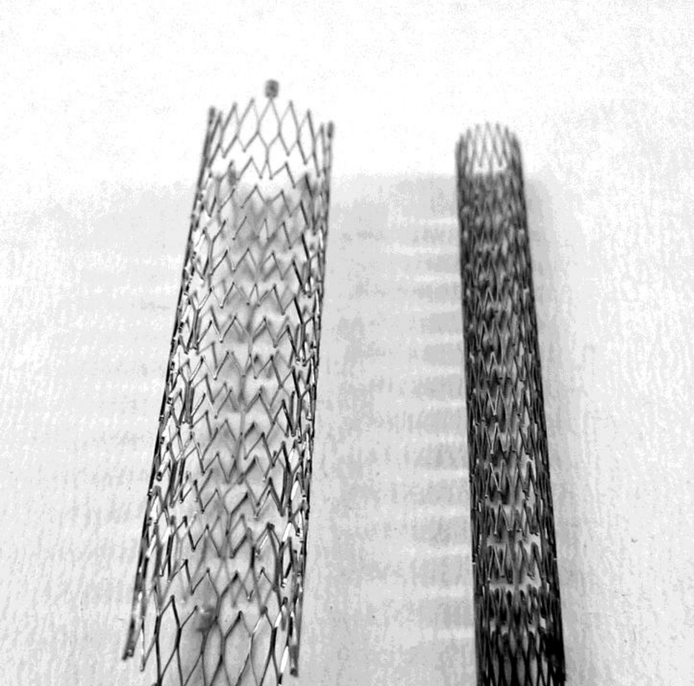


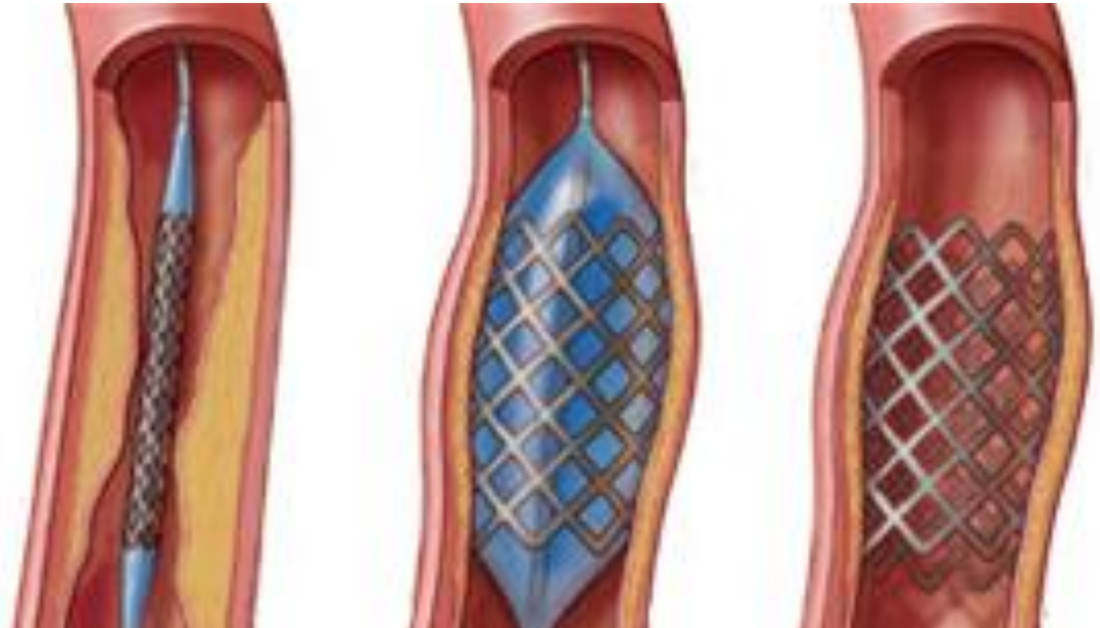

Workpiece
C
A
f
(axial – $\sigma_{\perp}$)
a_p
radial
B
V_c
(hoop – $\sigma_{\parallel}$)

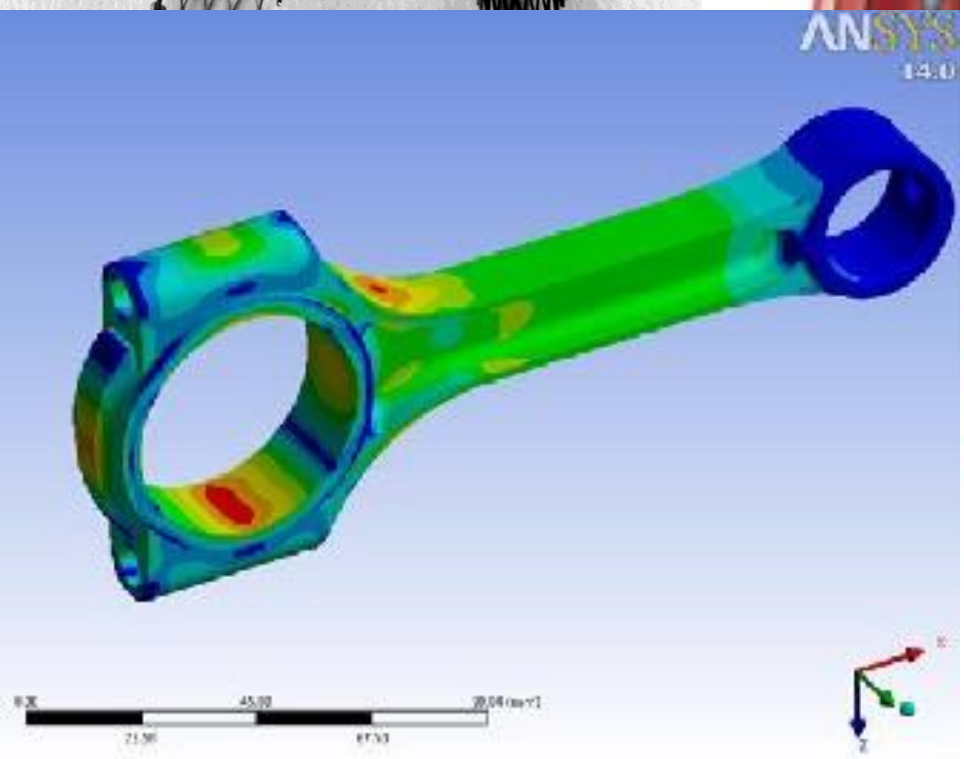

Four
Bar
Hinge
ACL
0°
PCL
90°
The joining of the femur and tibia bones with the two cruciate ligaments ACL & PCL forms a very sophisticated mechanism known as a four-bar hinge

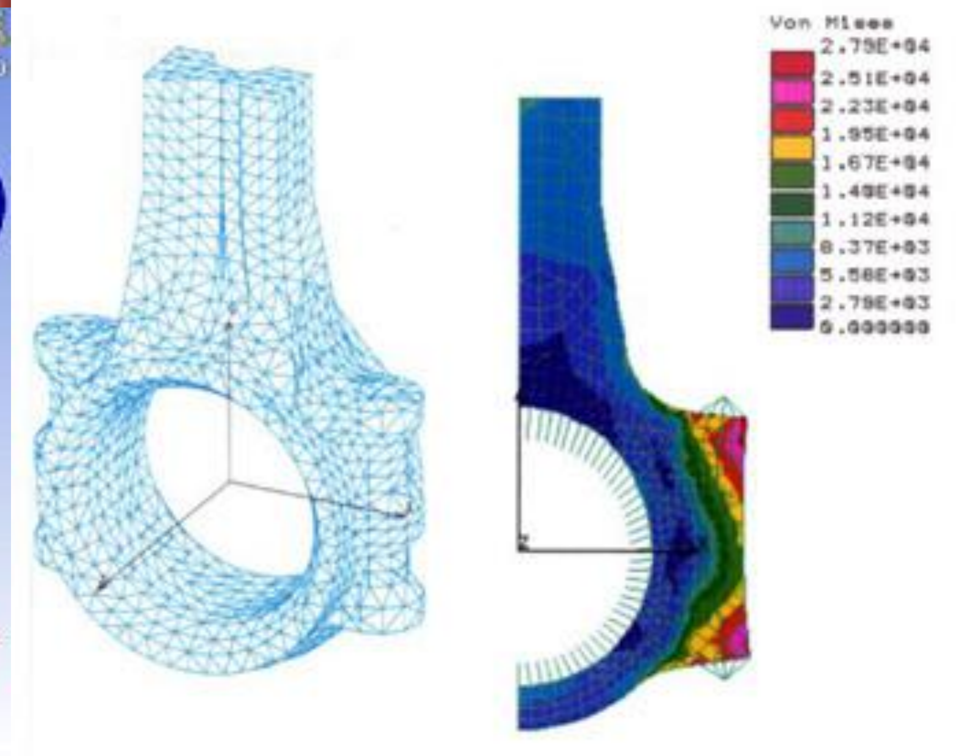

A: Static Structural (ANSYS)
Total Deformation
Type: Total Deformation

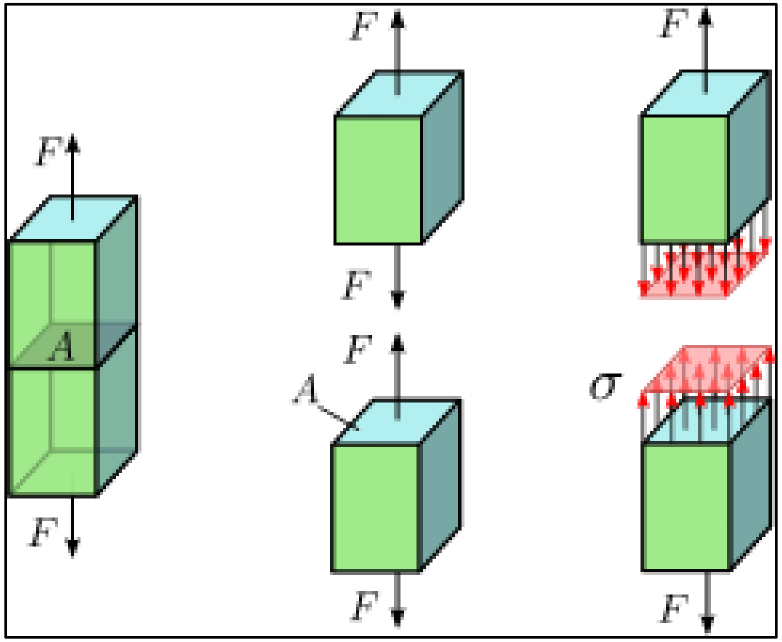

Buttress plate
A
B
C
D
E
Neutralization plate (with lag screw)
Bridge plate

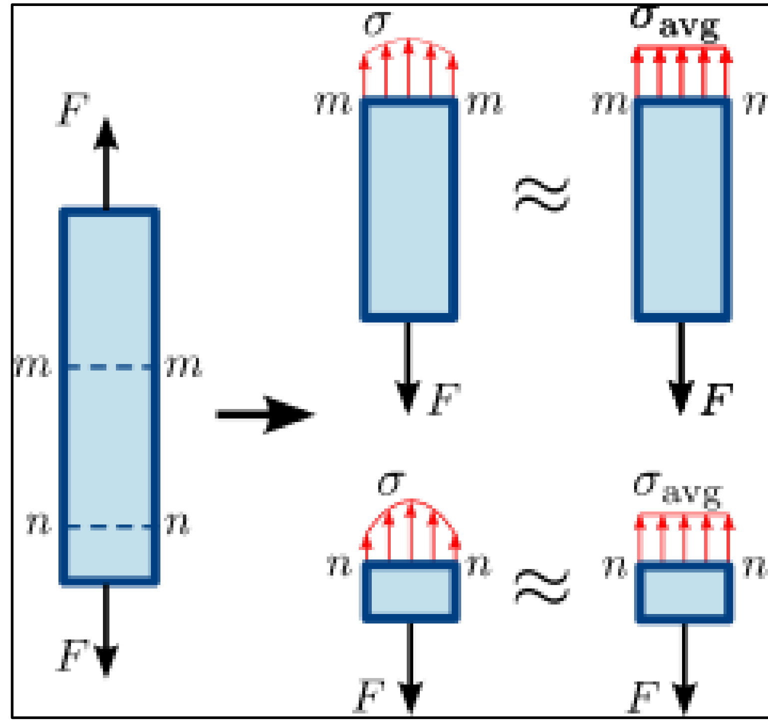


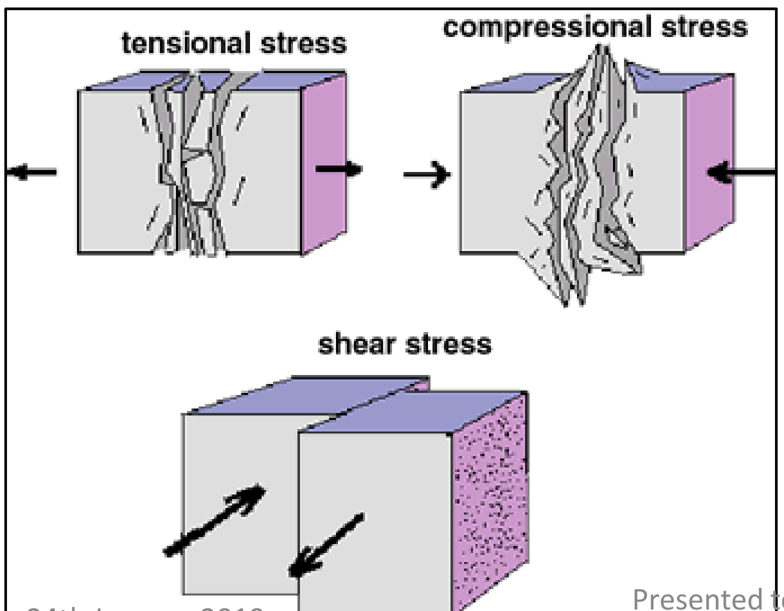



ANSYS
14.0


Von Mises
2.79E+04
2.51E+04
2.23E+04
1.95E+04
1.67E+04
1.40E+04
1.12E+04
8.37E+03
5.58E+03
2.79E+03
0.000000

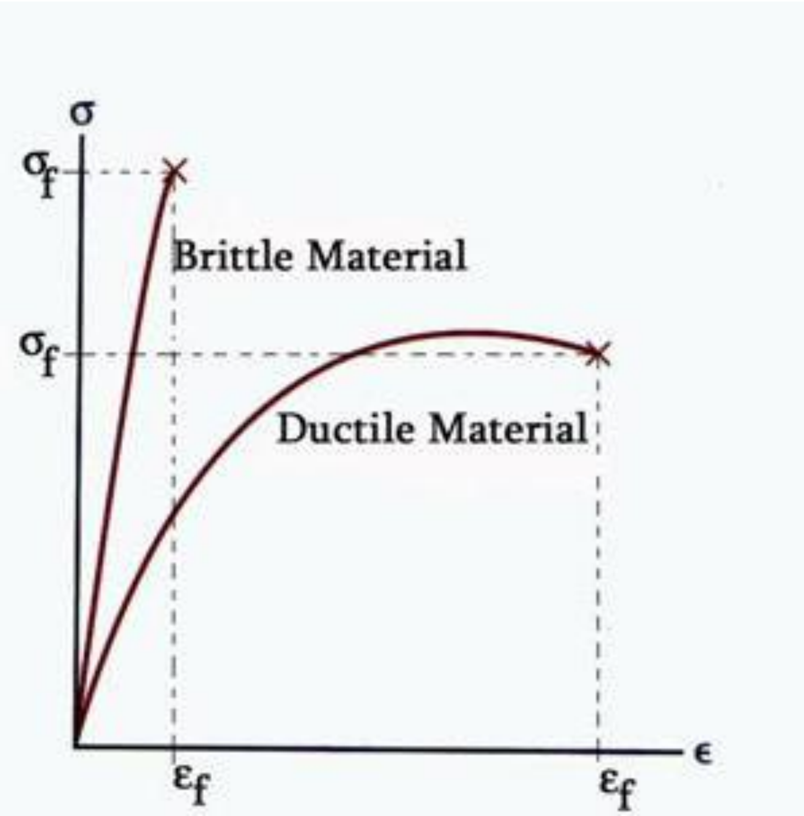




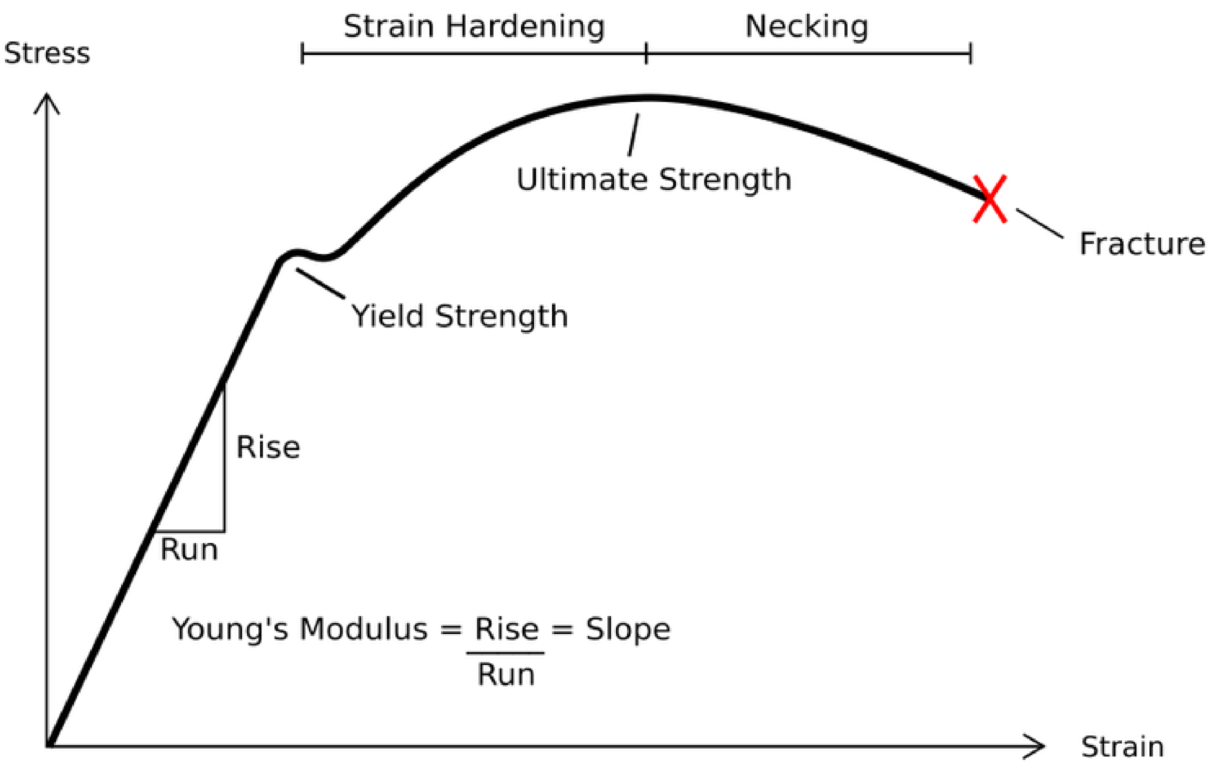


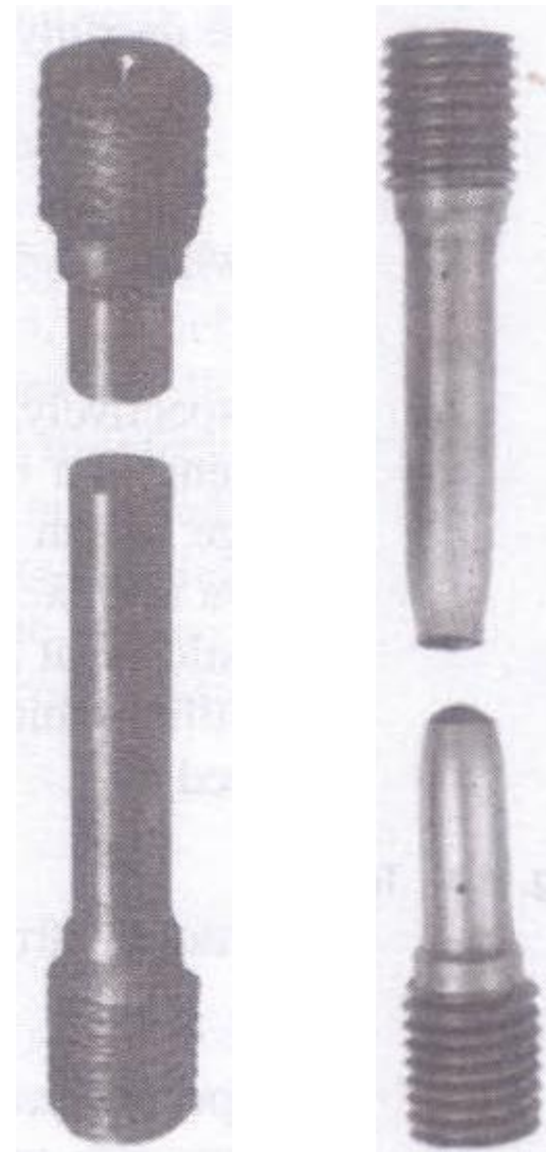
<u>TYPES OF STRESSES</u>

TENSION TEST:

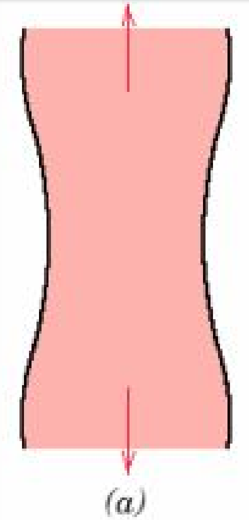


DUCTILE FAILURE

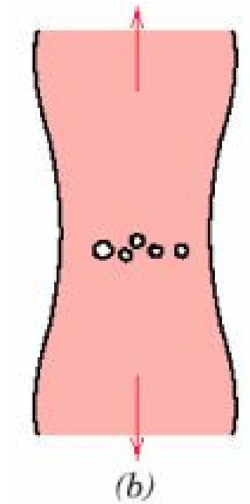




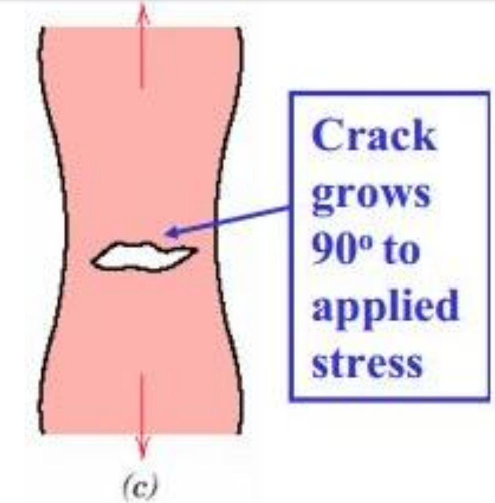
CUP AND CONE FORMATION:



Necking

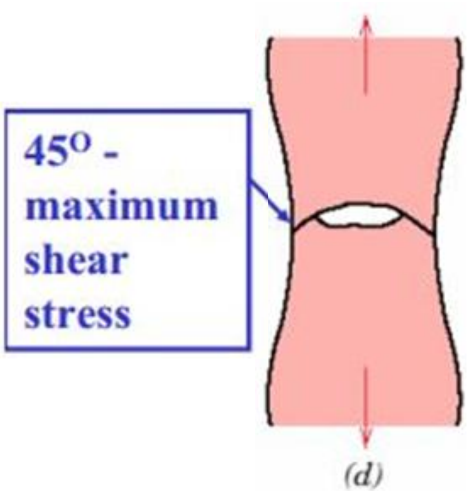


Formation of Microvoids

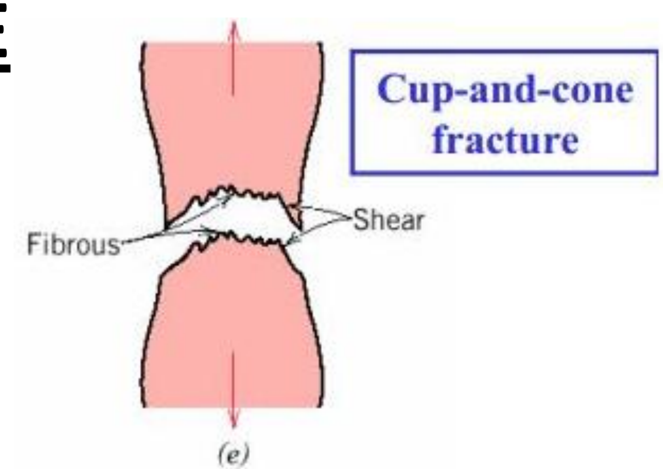


Coalescence of microvoids to form cracks

DUCTILE FRACTURE

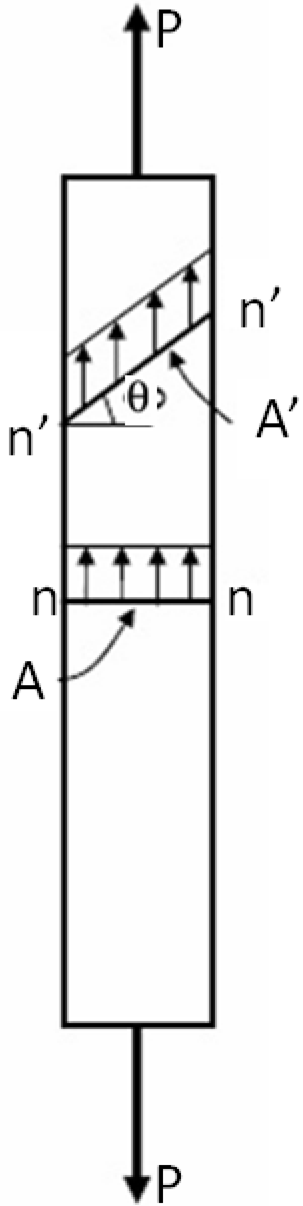


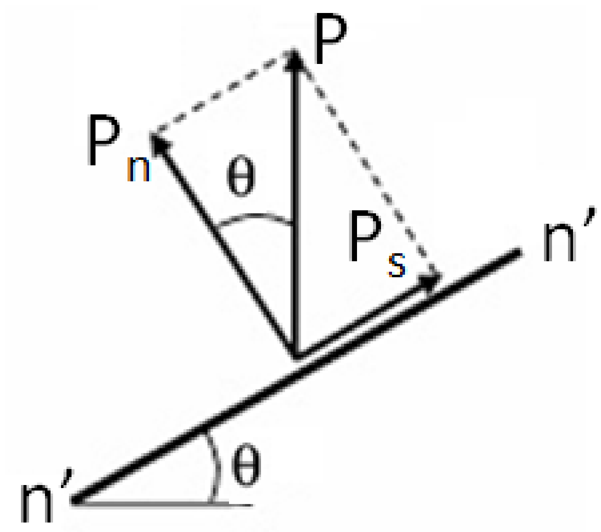
Crack Propogation by Shear deformation



Fracture

NATURE OF STRESS ON AN INCLINED PLANE:





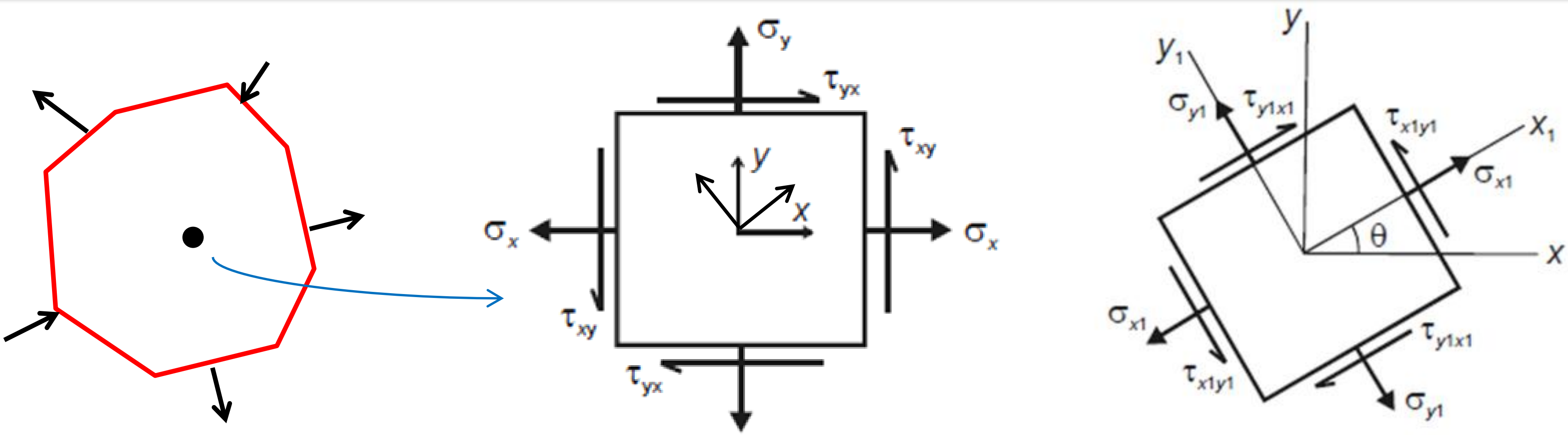
On Plane nn normal to the Applied Load:

$$\sigma = \frac{P}{A} \qquad \tau = 0$$

On Plane n'n' inclined at an angle θ to the applied load:

$$\sigma = \frac{P_n}{A'} = \frac{P\cos^2\theta}{A} \qquad \tau = \frac{P_s}{A'} = \frac{P.\sin\theta\cos\theta}{A}$$

TRANSFORMATION EQU. IN PLANE STRESS:

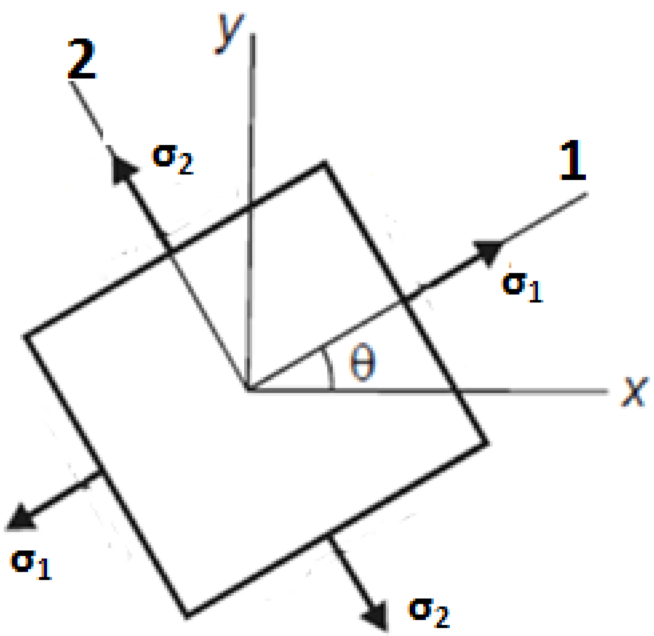


$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \mathrm{Cos}2\theta + \tau_{xy}\mathrm{Sin}2\theta$$

$$\tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2} \mathrm{Sin}2\theta + \tau_{xy}\mathrm{Cos}2\theta$$

$$\sigma_{y1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \mathrm{Cos}2\theta - \tau_{xy}\mathrm{Sin}2\theta$$

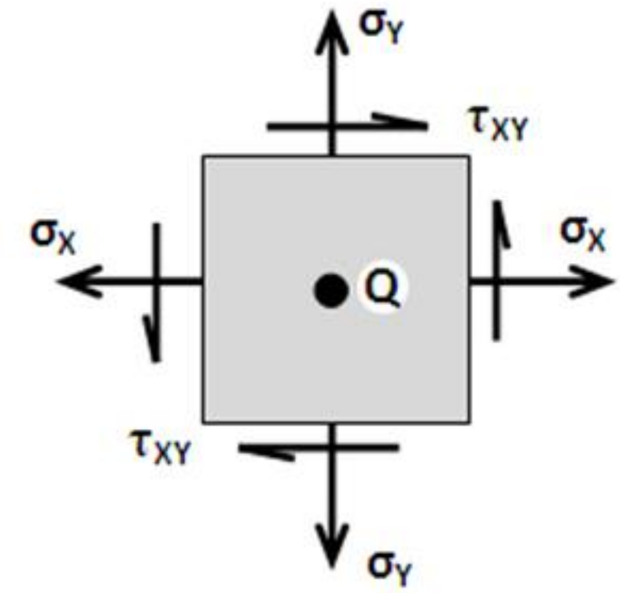
PRINCIPAL PLANE IN PLANE STRESS:



$$Tan2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

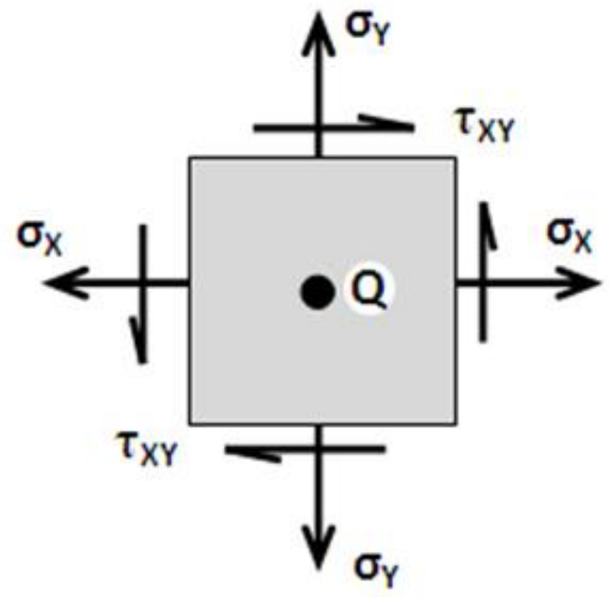
PLANES OF MAX. SHEAR IN PLANE STRESS (2D)



$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

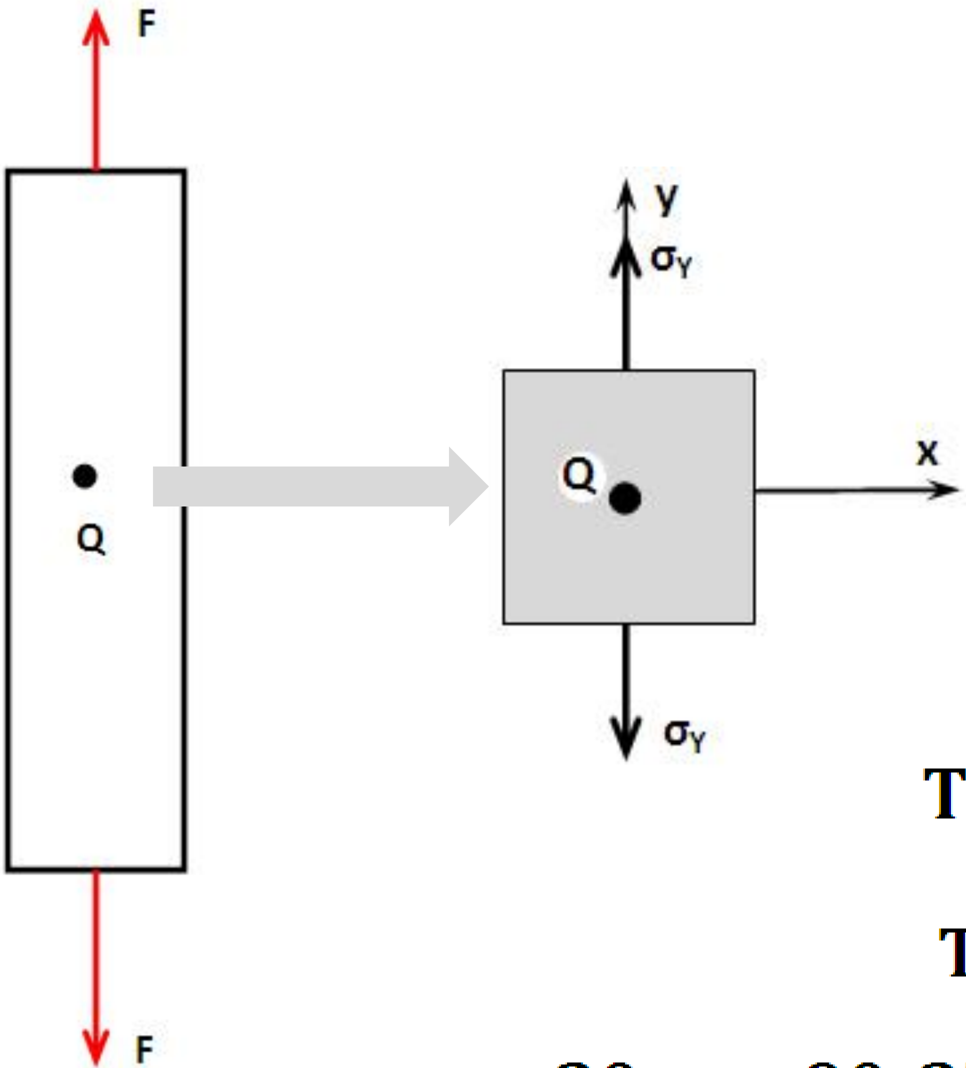
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Find the normal stress & shear stress on a 22.5^{O} plane and also the principal stresses, principal planes, max shear stress and planes of max. shear stress for the following states of stress.



1 σ_X = -60MPa σ_y = 0 MPa
τ_{XY} = 90 MPa

2 σ_X = 45MPa σ_y = 27MPa
τ_{XY} = 18MPa



$$\sigma_X = \tau_{XY} = 0$$

$$\text{Tan } 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{Tan } 2\theta_P = 0$$

$$2\theta_P = 0, 180$$

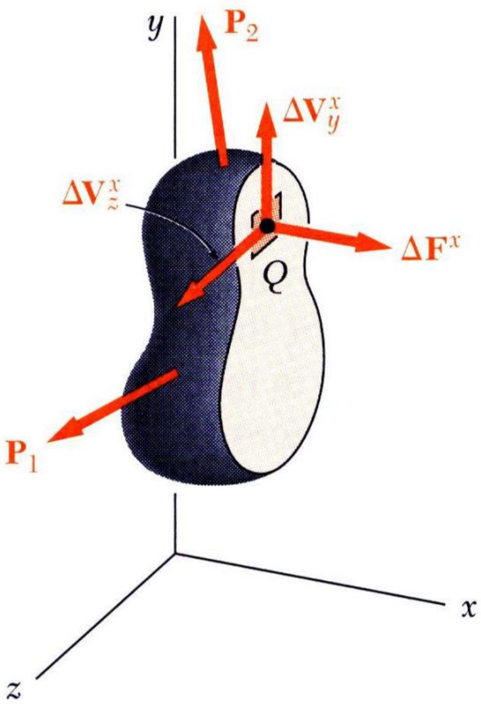
$$\theta_P = 0, 90$$

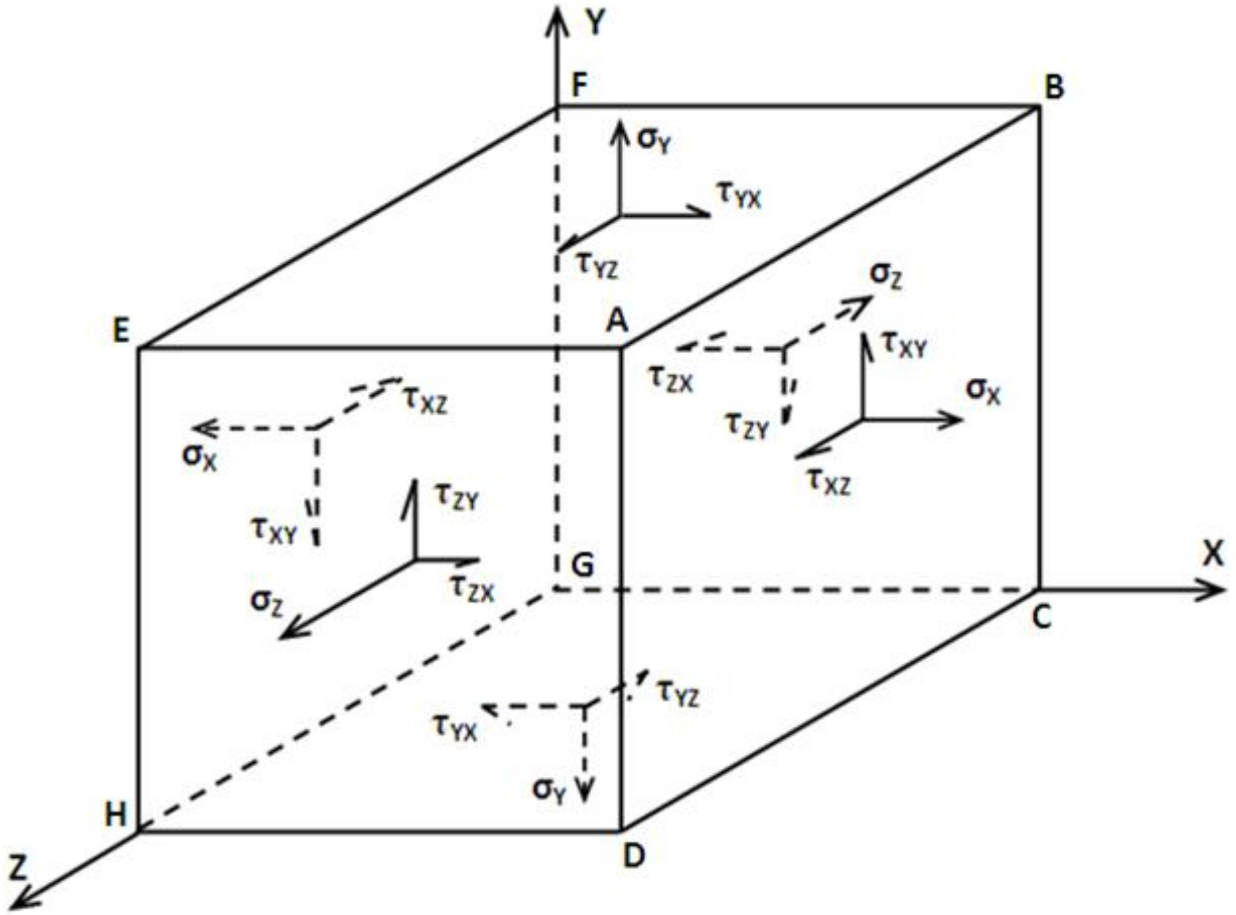
$$\text{Tan } 2\theta_S = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\text{Tan } 2\theta_S = \infty$$

$$2\theta_s = 90, 270 \quad \theta_s = 45, 135$$

3D STATE OF STRESS





Stress at a point is denoted by the stress tensor as given below:

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \quad Or \quad \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$$

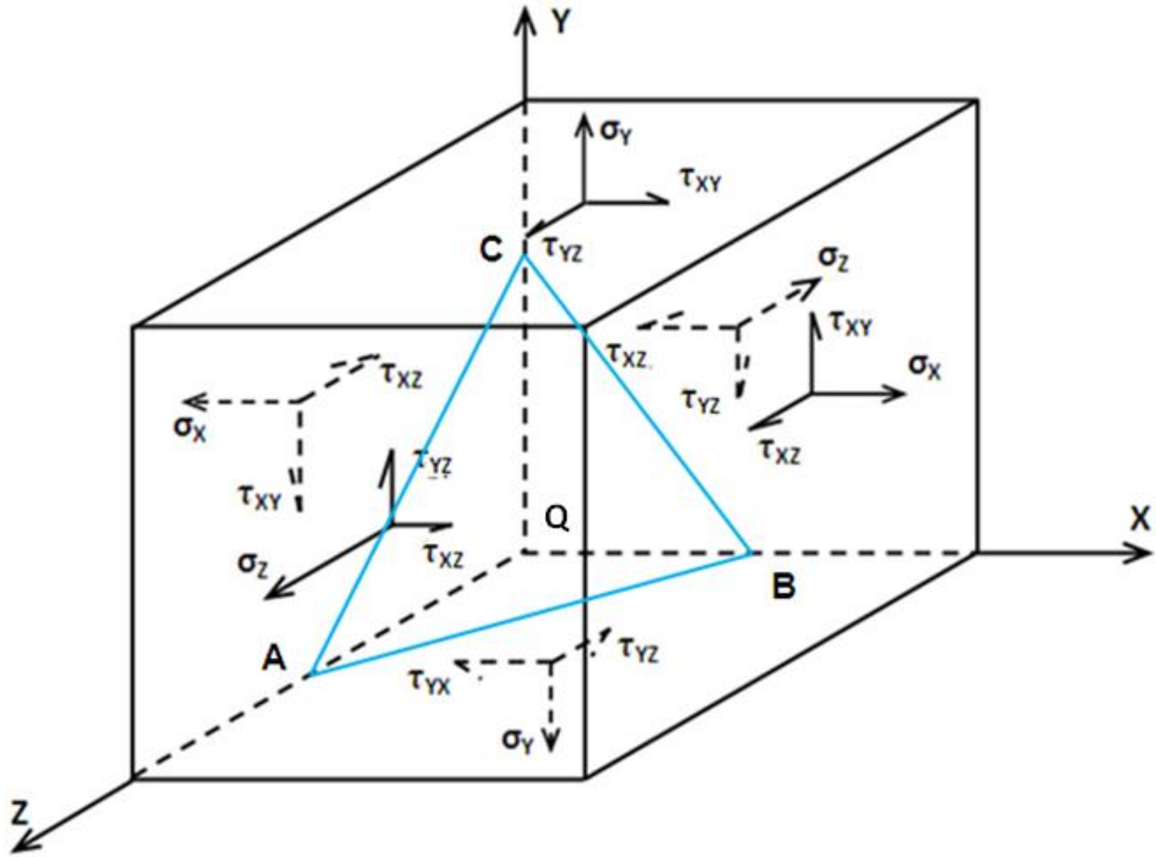
$$Or \quad \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{y} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{z} \end{bmatrix}$$

Consider a 3 D state of stress.

ABC is an arbitrary plane whose normal is n.

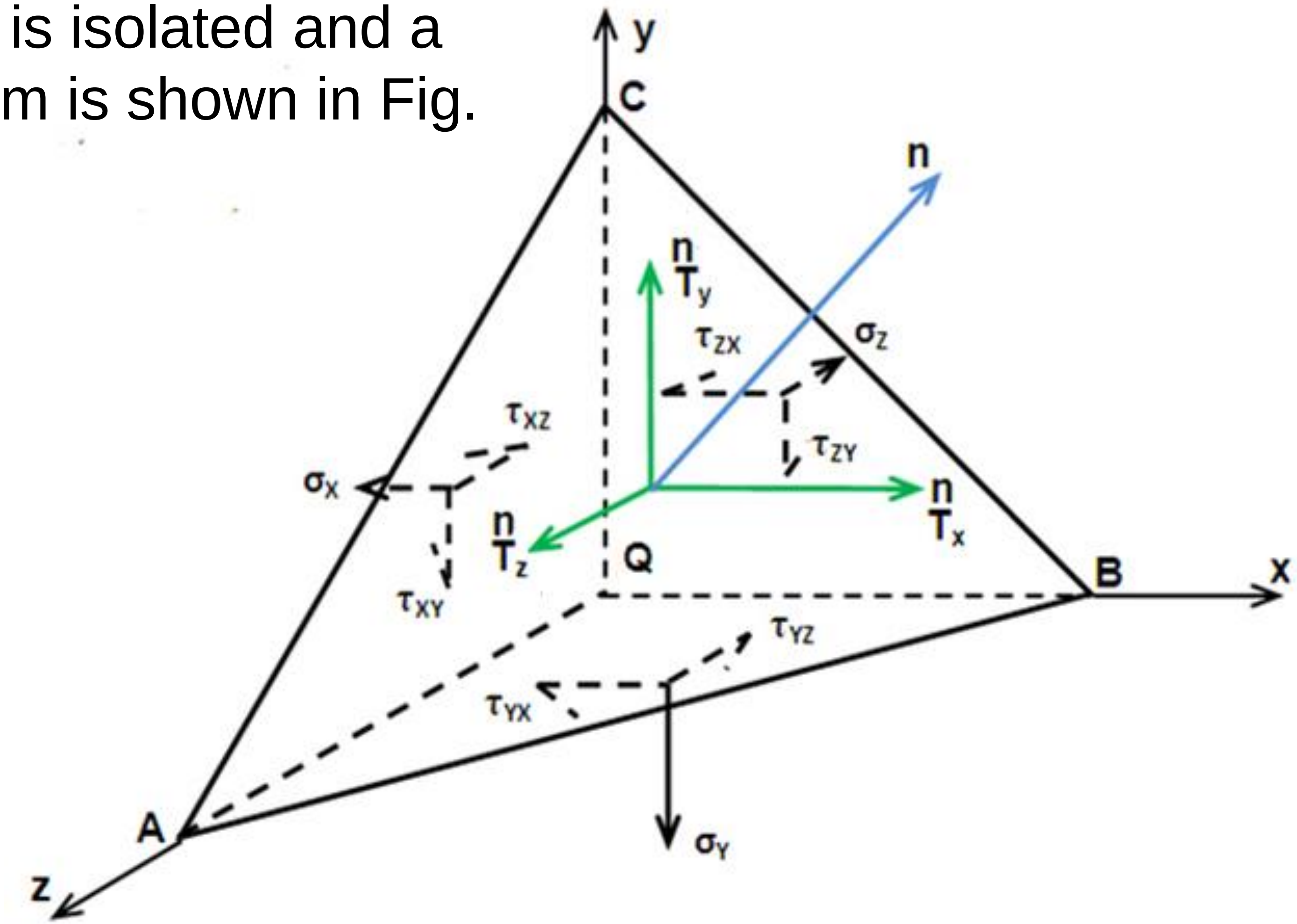
Direction Cosines of n are n_x, n_y and n_z.

Plane ABC is at a distance of **h** from Q



ABCD forms a tetrahedron.

The tetrahedron is isolated and a free body diagram is shown in Fig.



T^n - **Resultant stress vector on the plane.**

T_x^n - **Component along x axis**

T_y^n - **Component along y axis**

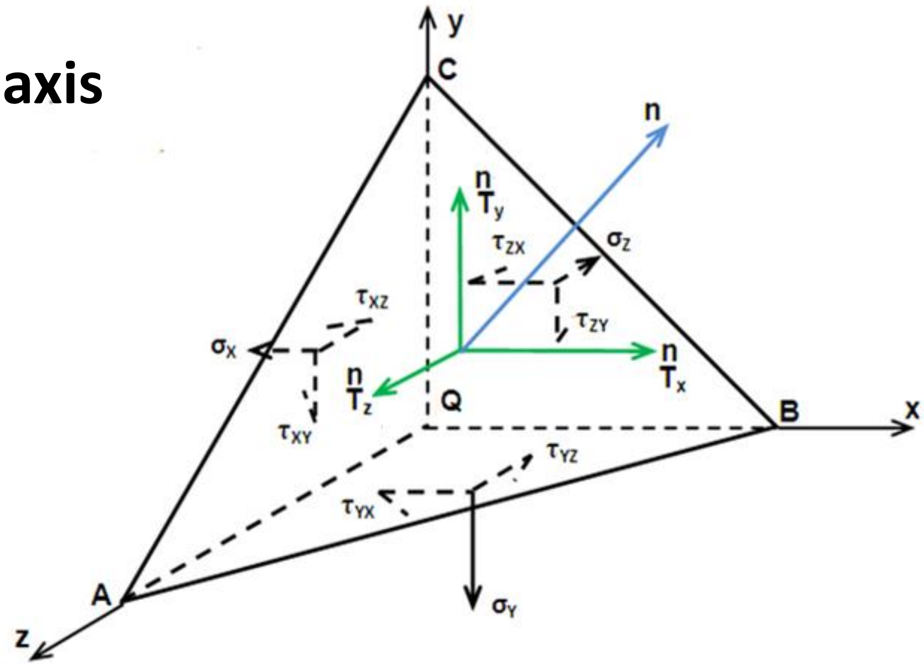
T_z^n - **Component along z axis**

A - Area of Plane ABC

Area of plane AQC = $\mathbf{A\ n_x}$

Area of plane AQB = $\mathbf{A\ n_y}$

Area of plane BQC = $\mathbf{A\ n_z}$



B_X , B_y , B_z – Body forces along x, y and z directions.

Volume of the tetrahedron = 1/3 Ah

Considering the equilibrium along the x, y and z axis we get,

$$T_x^n A = \sigma_x A n_x + \tau_{xy} A n_y + \tau_{xz} A n_z - B_x \frac{1}{3} Ah$$

Cancelling all A's and taking limit h-> 0, gives

$$T_x^n = \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z$$

Similarly considering equilibrium along y and z axis gives

$$T_y^n = \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z$$

$$T_z^n = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z$$

(1)

The above three equations are known as Cauchy's Stress equations.

Cauchy's stress equation can be written in the matrix form as

$$\begin{Bmatrix} T_x^n \\ T_y^n \\ T_z^n \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

The resultant stress vector on plane n is

$$|T^n|^2 = {T_x^n}^2 + {T_y^n}^2 + {T_z^n}^2 \qquad \text{------------} \qquad 2$$

The normal stress and shear stress on plane n can be obtained using the following equations

$$\sigma_n = n_x T_x^n + n_y T_y^n + n_z T_z^n \qquad \text{------------} \qquad 3$$

$$|T^n|^2 = {\sigma_n}^2 + {\tau_n}^2 \qquad \text{------------} \qquad 4$$

At a point Q in a body

σ_x = 10000 N/cm^2; σ_y = -5000 N/cm^2; σ_z = -5000 N/cm^2 τ_{xy} = τ_{xz} = τ_{xz} = 10000N/cm^2

Determine the normal and shear stress on a plane that is equally inclined to all three axis

$$T_x^n = 17320.5\ N/cm^2$$

$$T_y^n = 8660.25\ N/cm^2$$

$$T_x^n = 8660.25\ N/cm^2$$

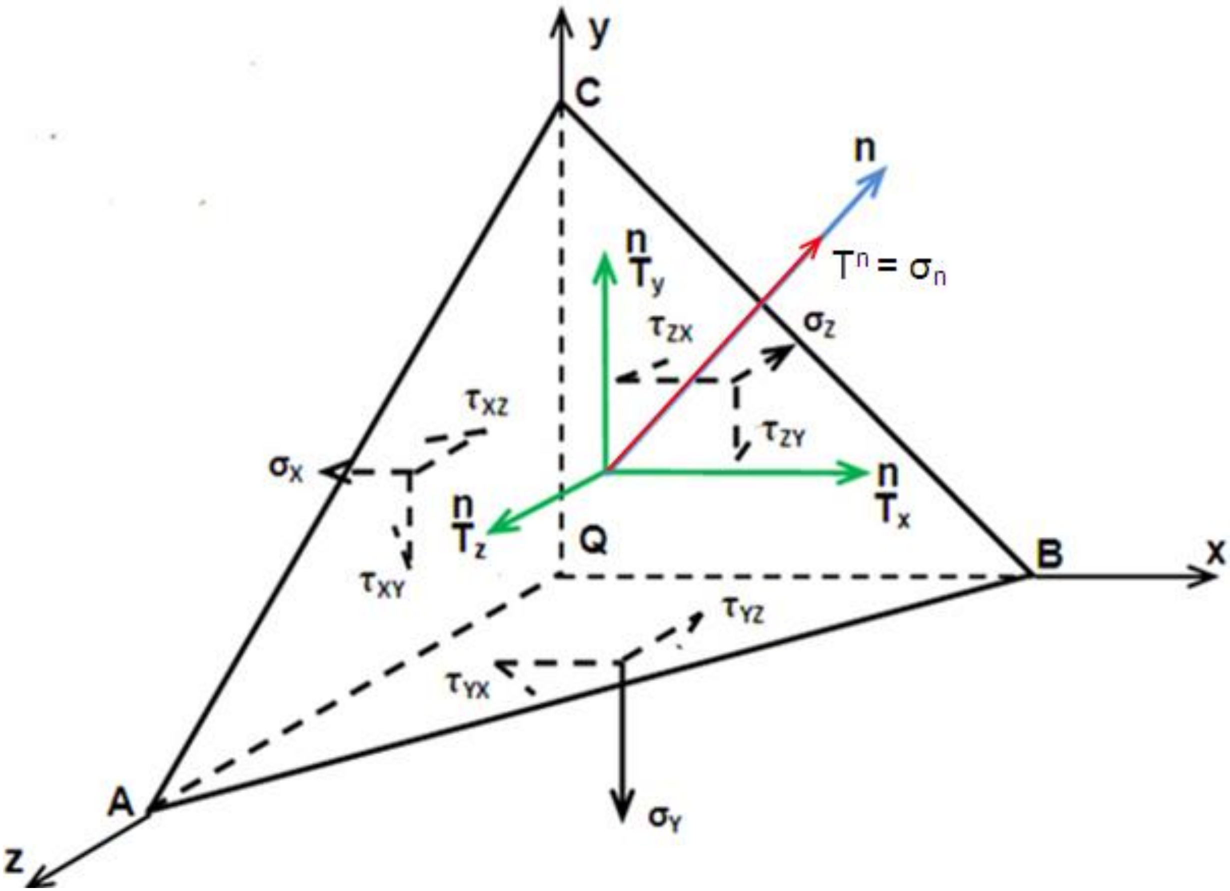
$$\sigma_n = 20000\ \mathrm{N/cm^2}$$

$$|T^n|^2 = 450 \times 10^6$$

$$\tau^n = 7071\ \mathrm{N/cm^2}$$

PRINCIPAL STRESSES IN 3D STATE OF STRESS

A Plane where there is no shear stress is called a Principal Plane.



PRINCIPAL STRESSES IN 3D STATE OF STRESS

Let n_x, n_y and n_z be the Direction Cosines of the Principal Plane.

$$\left.\begin{aligned} T_x^n &= \sigma . n_x \\ T_y^n &= \sigma . n_y \\ T_z^n &= \sigma . n_z \end{aligned}\right\} \quad 1$$

where, σ is the Principal Stress.

Using Cauchy's Equation,

$$\left.\begin{aligned} T_x^n &= \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z \\ T_y^n &= \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z \\ T_z^n &= \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z \end{aligned}\right\} \quad 2$$

PRINCIPAL STRESSES IN 3D STATE OF STRESS

Equating Eqs. 1 and 2 we get

$$\sigma . n_x = \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z$$

$$\sigma . n_y = \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z$$

$$\sigma . n_z = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z$$

$$\left.\begin{aligned} (\sigma_x - \sigma)\, n_x + \tau_{xy} n_y + \tau_{xz} n_z &= 0 \\ \tau_{xy} n_x + (\sigma_y - \sigma) n_y + \tau_{yz} n_z &= 0 \\ \tau_{xz} n_x + \tau_{yz} n_y + (\sigma_z - \sigma)\, n_z &= 0 \end{aligned}\right\} \quad 3$$

PRINCIPAL STRESSES IN 3D STATE OF STRESS

$$\begin{vmatrix} (\sigma_x - \sigma) & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & (\sigma_y - \sigma) & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - \sigma) \end{vmatrix} = 0$$

Expanding the above equation we get

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - {\tau_{xy}}^2 - {\tau_{xz}}^2 - {\tau_{yz}}^2)\sigma - (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \sigma_x{\tau_{yz}}^2 - \sigma_y{\tau_{xz}}^2 - \sigma_z{\tau_{xy}}^2) = 0$$

PRINCIPAL STRESSES IN 3D STATE OF STRESS

$$\sigma^3 - I_1\,\sigma^2 + I_2\sigma - I_3 = 0$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{yz} & \sigma_z \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix}$$

Principal Stresses can be found out by solving the above cubical equation.

INVARIANTS OF STRESS

I_1, I_2 and I_3 are called Stress Invariants.

They are called so because the values of I_1, I_2, I_3 does not change even if the reference co ordinates are changed. In the cubical equ.

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

I_1 - First Stress Invariant

I_2 - Second Stress Invariant

I_3 - Third Stress Invariant

INVARIANTS OF STRESS

Let x', y', z' be another frame of reference at the same point. With respect to the frame of reference the stress state is given by,

$$\begin{bmatrix} \sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\ \tau_{y'x'} & \sigma_{y'} & \tau_{y'z'} \\ \tau_{z'x'} & \tau_{z'y'} & \sigma_{z'} \end{bmatrix}$$

INVARIANTS OF STRESS

$$I_1^{\,l} = \sigma_{x'} + \sigma_{y'} + \sigma_{z'}$$

$$I_2^{\,l} = \begin{vmatrix} \sigma_{x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'} \end{vmatrix} + \begin{vmatrix} \sigma_{x'} & \tau_{x'z'} \\ \tau_{x'z'} & \sigma_{z'} \end{vmatrix} + \begin{vmatrix} \sigma_{y'} & \tau_{y'z'} \\ \tau_{y'z'} & \sigma_{z'} \end{vmatrix}$$

$$I_3^{\,l} = \begin{vmatrix} \sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\ \tau_{x'y'} & \sigma_{y'} & \tau_{y'z'} \\ \tau_{x'z'} & \tau_{y'z'} & \sigma_{z'} \end{vmatrix}$$

INVARIANTS OF STRESS

The principal stresses at a point depends only on the load exerted on the body and not on the co ordinates of reference describing the rectangular stress components hence,

$$\sigma^3 - I_1 \sigma^2 + I_2\sigma - I_3 = 0$$

$$\sigma^3 - I_1{}^l \sigma^2 + I_2{}^l\sigma - I_3{}^l = 0$$

must give same solutions for σ. So the coefficients σ^2, σ and constant term in the two equs. must be equal. Thus

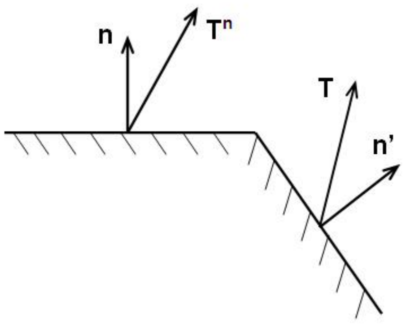
$$I_1 = I_1^l;\ I_2 = I_2^l;\ I_3 = I_3^l$$

INVARIANTS OF STRESS

Find the principal stresses and their planes for the following state of stress

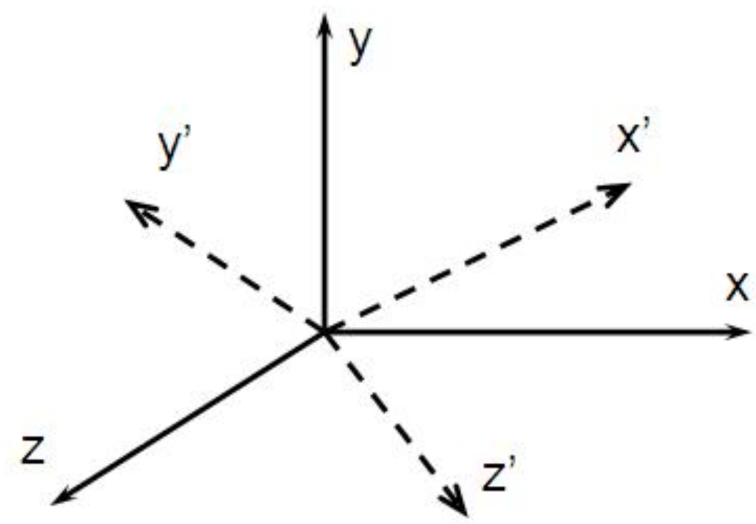
$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & -3 \\ 1 & -3 & 4 \end{bmatrix}$$

Theorem 1: If n and n' are two planes through same point P with corresponding stress vectors T^n and $T^{n'}$ Then the projection of T^n along n' is equal to the projection of $T^{n'}$ along n



Theorem 2: Principal planes are orthogonal.

STRESS TRANSFORMATION



$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\ \tau_{y'x'} & \sigma_{y'} & \tau_{y'z'} \\ \tau_{z'x'} & \tau_{z'y'} & \sigma_{z'} \end{bmatrix}$$

STRESS TRANSFORMATION

Direction Cosines of x' be $n_{xx'}$, $n_{yx'}$, $n_{zx'}$

Direction Cosines of y' be $n_{xy'}$, $n_{yy'}$, $n_{zy'}$

Direction Cosines of z' be $n_{xz'}$, $n_{yz'}$, $n_{zz'}$

$n_{xx'}$ – Cos of angle between x and x'

$n_{yx'}$ - Cos of angle between y and x'

$n_{zx'}$ - Cos of angle between z and x'

STRESS TRANSFORMATION

While taking the sign of angle in xy plane anticlockwise direction is taken as positive while looking to the xy plane from the +ve z axis.

According to Caushy's equation

$$\left.\begin{aligned} T_x^{x^l} &= \sigma_x n_{xx'} + \tau_{xy} n_{yx'} + \tau_{xz} n_{zx'} \\ T_y^{x^l} &= \tau_{xy} n_{xx'} + \sigma_y n_{yx'} + \tau_{yz} n_{zx'} \\ T_z^{x^l} &= \tau_{xz} n_{xx'} + \tau_{yz} n_{yx'} + \sigma_z n_{zx'} \end{aligned}\right\} \quad (1)$$

STRESS TRANSFORMATION

For getting the component of $T^{x'}$ along the x' direction take the dot product of $T^{x'}$ and x'

For getting the component of $T^{x'}$ along the y' direction take the dot product of $T^{x'}$ and y'

For getting the component of $T^{x'}$ along the z' direction take the dot product of $T^{x'}$ and z'

$$\left.\begin{aligned} \sigma_{x'} &= T_x^{x'} n_{xx'} + T_y^{x'} n_{yx'} + T_z^{x'} n_{zx'} \\ \tau_{x'y'} &= T_x^{x'} n_{xy'} + T_y^{x'} n_{yy'} + T_z^{x'} n_{zy'} \\ \tau_{x'z'} &= T_x^{x'} n_{xz'} + T_y^{x'} n_{yz'} + T_z^{x'} n_{zz'} \end{aligned}\right\} \quad (2)$$

STRESS TRANSFORMATION

Substituting for $T_x^{x'}$ $T_y^{x'}$ $T_z^{x'}$ from equ 1 in equ 2 will give,

$$\sigma_{x'x'} = \sigma_{xx} n_{xx'}^2 + \sigma_{yy} n_{yx'}^2 + \sigma_{zz} n_{zx'}^2 + 2\tau_{xy} n_{xx'} n_{yx'} + 2\tau_{xz} n_{xx'} n_{zx'} + 2\tau_{yz} n_{yx'} n_{zx'}$$

$$\tau_{x'y'} = \sigma_{xx} n_{xx'} n_{xy'} + \sigma_{yy} n_{yx'} n_{yy'} + \sigma_{zz} n_{zx'} n_{zy'} + \tau_{xy}\left(n_{xx'} n_{yy'} + n_{xy'} n_{yx'}\right) + \tau_{yz}\left(n_{yx'} n_{zy'} + n_{zx'} n_{yy'}\right) + \tau_{xz}\left(n_{xx'} n_{zy'} + n_{zx'} n_{xy'}\right)$$

$$\tau_{x'z'} = \sigma_{xx} n_{xx'} n_{xz'} + \sigma_{yy} n_{yx'} n_{yz'} + \sigma_{zz} n_{zx'} n_{zz'} + \tau_{xy}\left(n_{xx'} n_{yz'} + n_{yx'} n_{xz'}\right) + \tau_{yz}\left(n_{yx'} n_{zz'} + n_{zx'} n_{yz'}\right) + \tau_{xz}\left(n_{xx'} n_{zz'} + n_{zx'} n_{xz'}\right)$$

The above set of three equations can be written in the matrix form as

$$\begin{Bmatrix} \sigma_{x'x'} \\ \tau_{x'y'} \\ \tau_{x'z'} \end{Bmatrix} = \begin{bmatrix} n_{xx'} & n_{yx'} & n_{zx'} \\ n_{xy'} & n_{yy'} & n_{zy'} \\ n_{xz'} & n_{yz'} & n_{zz'} \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{Bmatrix} n_{xx'} \\ n_{yx'} \\ n_{zx'} \end{Bmatrix}$$

$$\{\sigma\}_{x'} = [\alpha]^T [\sigma]_{xyz} \{n\}_{x'} \qquad \text{-----------} \qquad (3)$$

$\{\sigma\}_{x'}$ - **Stress components on x' plane**

$[\sigma]_{xyz}$ - **Stress components on xyz plane**

$\{n\}_{x'}$ - **Direction Cosines of x'**

STRESS TRANSFORMATION

$$[\alpha] = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} (x') & (y') & (z') \\ \end{array}$$

$$[\alpha] = \begin{bmatrix} n_{xx'} & n_{xy'} & n_{xz'} \\ n_{yx'} & n_{yy'} & n_{yz'} \\ n_{zx'} & n_{zy'} & n_{zz'} \end{bmatrix} \begin{matrix} (x) \\ (y) \\ (z) \end{matrix}$$

$$\{\sigma\}_{y'} = [\alpha]^T [\sigma]_{xyz} \{n\}_{y'} \quad \text{-----------} \quad \textbf{(4)}$$

The Stresses on the z' plane is obtained as,

$$\{\sigma\}_{z'} = [\alpha]^T [\sigma]_{xyz} \{n\}_{z'} \quad \text{-----------} \quad \textbf{(5)}$$

The Stresses on the y' plane is obtained as,

STRESS TRANSFORMATION

Combining equs. 3,4 & 5

The Stress transformation equation is obtained:

$$\{\boldsymbol{\sigma}\}_{x'y'z'} = [\alpha]^T [\boldsymbol{\sigma}]_{xyz} [\alpha]$$

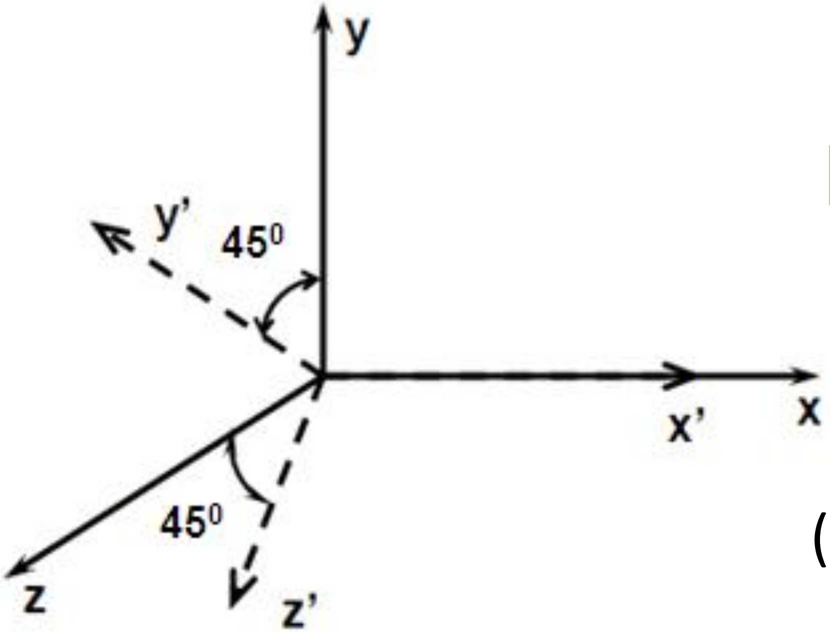
STRESS TRANSFORMATION

A state of stress at a point with respect to xyz is given by ,

$$\sigma = \begin{bmatrix} 10 & 6 & -8 \\ 6 & 20 & -4 \\ -8 & -4 & 10 \end{bmatrix} \text{MPa}$$

Find the state of stress for new set of axis rotated about x axis to an angle 45^0.

STRESS TRANSFORMATION



$$[\sigma]_{xyz} = \begin{bmatrix} 10 & 6 & -8 \\ 6 & 20 & -4 \\ -8 & -4 & 10 \end{bmatrix} \text{MPa}$$

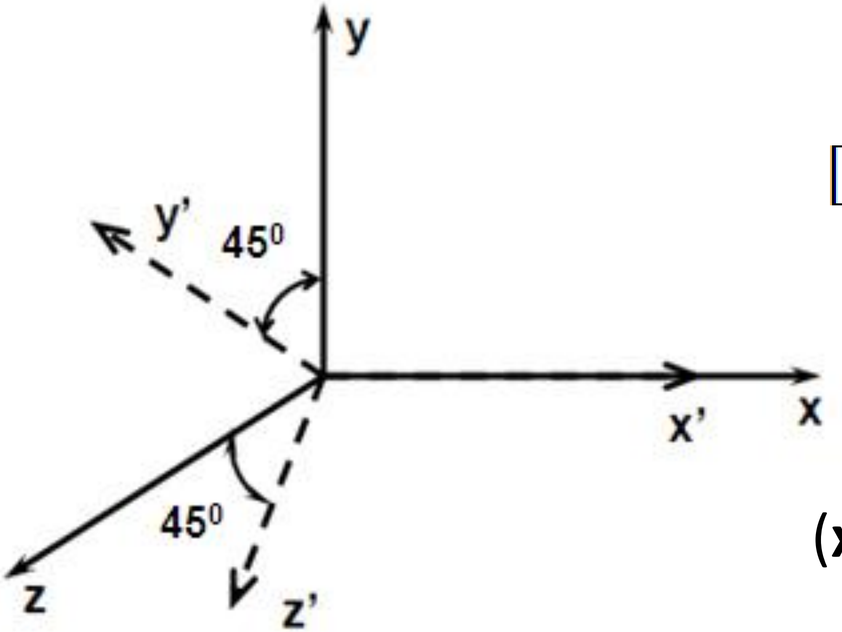
(x') (y') (z')

(x)

(y)

(z)

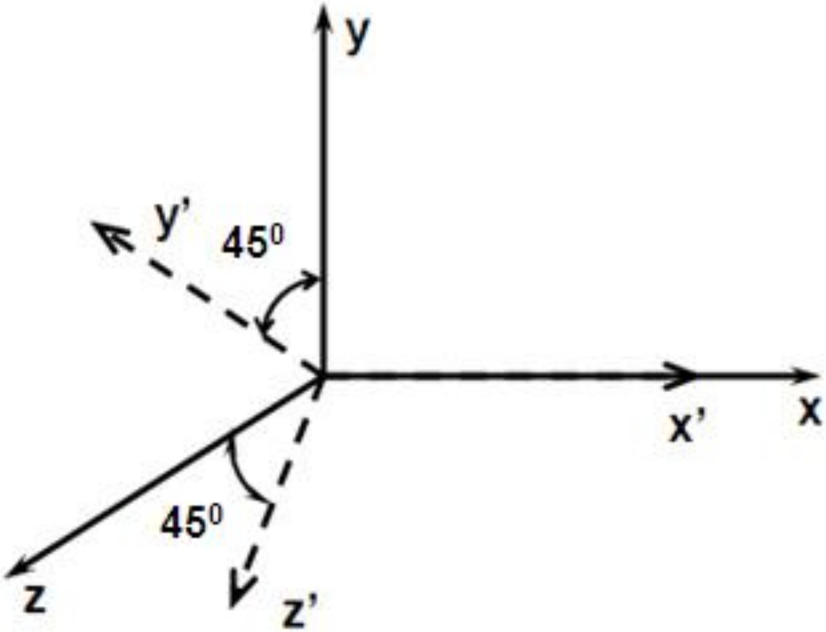
STRESS TRANSFORMATION



$$[\sigma]_{xyz} = \begin{bmatrix} 10 & 6 & -8 \\ 6 & 20 & -4 \\ -8 & -4 & 10 \end{bmatrix} \text{MPa}$$

$$\begin{matrix} & (x') & (y') & (z') & \\ [\alpha] = & \begin{bmatrix} \text{Cos}\,(0) & \text{Cos}\,(90) & \text{Cos}\,(-90) \\ \text{Cos}\,(-90) & \text{Cos}\,(45) & \text{Cos}\,(135) \\ \text{Cos}\,(90) & \text{Cos}\,(-45) & \text{Cos}\,(45) \end{bmatrix} & \begin{matrix} (x) \\ (y) \\ (z) \end{matrix} \end{matrix}$$

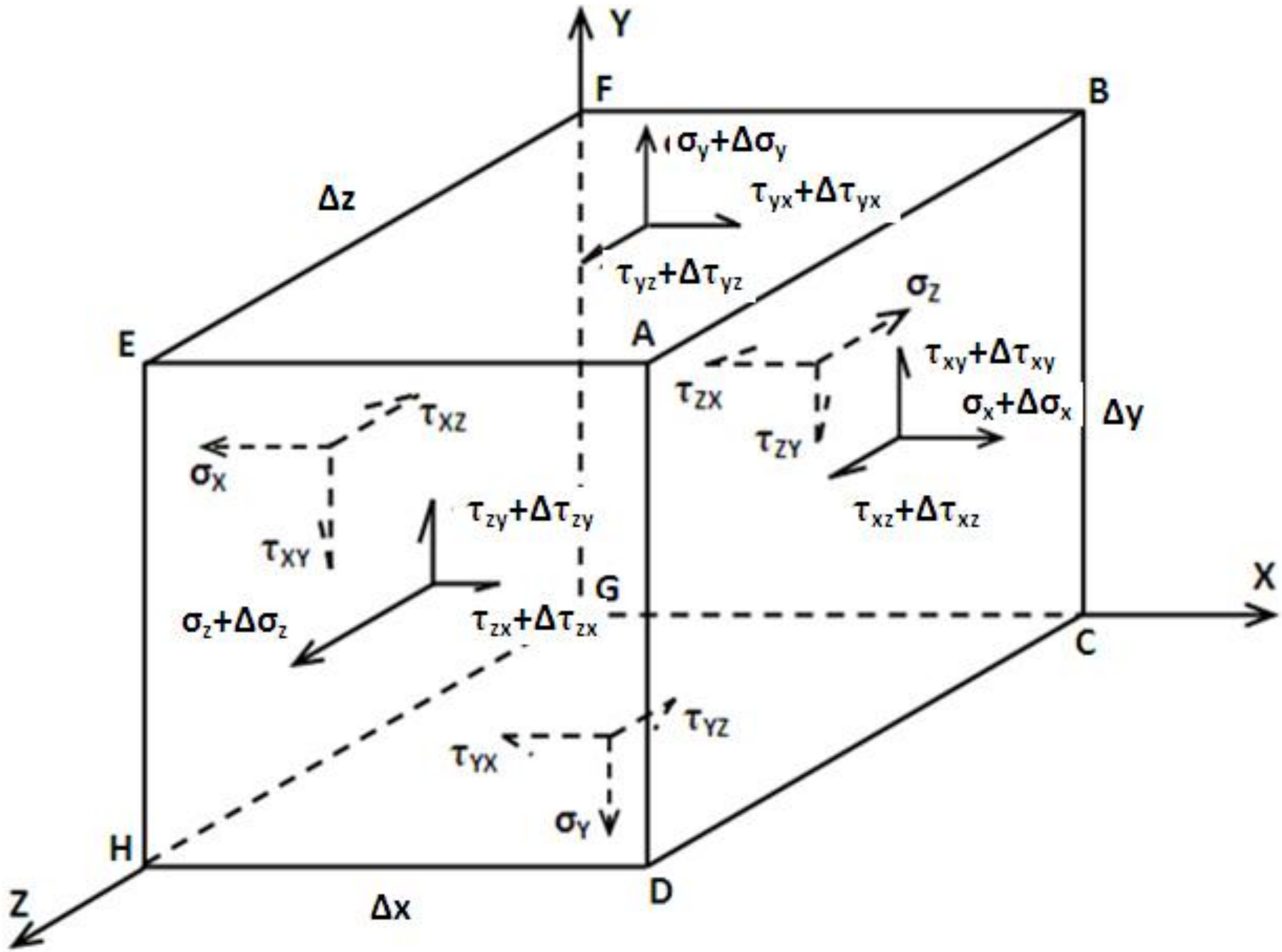
STRESS TRANSFORMATION



$$[\sigma]_{xyz} = \begin{bmatrix} 10 & 6 & -8 \\ 6 & 20 & -4 \\ -8 & -4 & 10 \end{bmatrix} \text{MPa}$$

$$\begin{matrix} & (x') & (y') & (z') & \\ [\alpha] = & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.7071 & -0.7071 \\ 0 & 0.7071 & 0.7071 \end{bmatrix} & & & \begin{matrix} (x) \\ (y) \\ (z) \end{matrix} \end{matrix}$$

DIFFERENTIAL EQUATION OF EQUILIBRIUM



DIFFERENTIAL EQUATION OF EQUILIBRIUM

Let the body force components per unit volume in the x y and z direction be B_X, B_Y and B_Z.

For equilibrium along x direction,

$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x}\Delta x\right)\Delta y \Delta z - \sigma_x \Delta y \Delta z + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)\Delta x \Delta z - \tau_{yx}\Delta x \Delta z + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\Delta z\right)\Delta x \Delta y - \tau_{zx}\Delta x \Delta y + B_x \Delta x \Delta y \Delta z = 0$$

DIFFERENTIAL EQUATION OF EQUILIBRIUM

Dividing by ΔxΔyΔz and taking the limits ΔxΔyΔz tends to zero

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x = 0$$

DIFFERENTIAL EQUATION OF EQUILIBRIUM

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + B_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + B_z = 0$$

Equilibrium Equation is also called differential equation of motion for a deformable body.

DIFFERENTIAL EQUATION OF EQUILIBRIUM

A cross section of wall of dam is showed in fig. The pressure of water on face OB is also shown in fig. The stress at any point xy are given below γ – Specific weight of water, ρ – specific weight of dam material.

$$\sigma_x = -\gamma y$$

$$\sigma_y = \left(\frac{\rho}{\tan\beta} - \frac{2\gamma}{\tan^3\beta}\right)x + \left(\frac{\gamma}{\tan^2\beta} - \rho\right)y$$

$$\tau_{xy} = -\frac{\gamma}{\tan^2\beta}x$$

HYDROSTATIC AND DEVIATORIC STATE OF STRESS

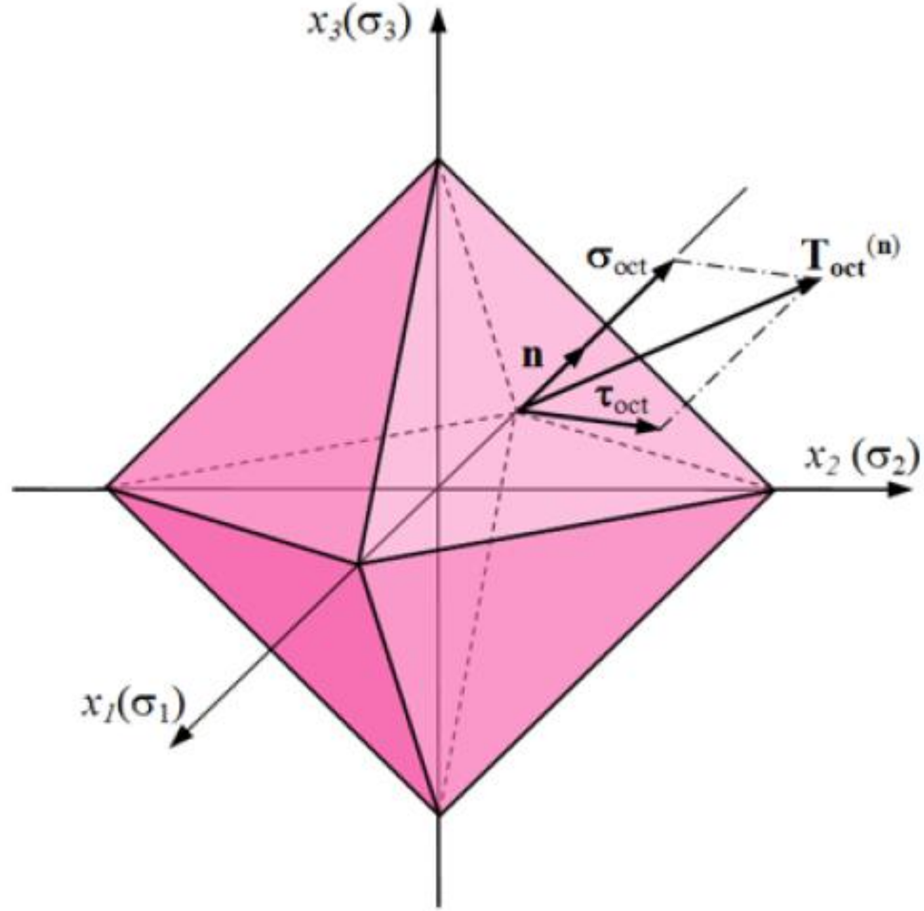
$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{bmatrix}}_{\text{Hydrostatic State.}} + \underbrace{\begin{bmatrix} \sigma_{xx} - P & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} - P & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} - P \end{bmatrix}}_{\text{Deviatoric state.}}$$

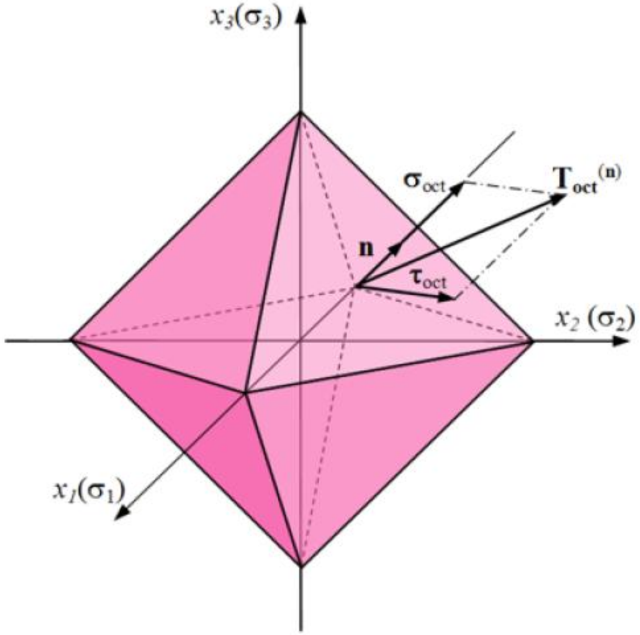
Where, $P = \frac{1}{3}\left[\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right] = \frac{1}{3} I_1$

OCTAHEDRAL STRESSES

Consider the principal directions as the coordinate axes. The plane whose normal vector forms equal angles with the coordinate system is called octahedral plane. There are eight such planes forming an octahedron.



OCTAHEDRAL STRESSES



$$[\sigma] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$n_1 = n_2 = n_3 = \frac{1}{\sqrt{3}}$$

$$\sigma_{oct} = \frac{1}{3}\,[\sigma_1 + \sigma_2 + \sigma_3] = \frac{1}{3} I_1$$

$$\tau_{oct} = \frac{1}{3}\sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = \frac{1}{3}\sqrt{2{I_1}^2 - 6I_2}$$

$$= \sqrt{\frac{2}{3} I'_2}$$

Where I’ is the second invariant of deviatoric stress tensor

Calculate the octahedral stresses for the following stress tensor:

$$\sigma_{ij} = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 3 & 4 \\ -1 & 4 & -2 \end{bmatrix}$$

Calculate the stress deviator tensor and its invariants for the following stress tensor:

$$\sigma_{ij} = \begin{bmatrix} 2 & -3 & 4 \\ -3 & -5 & 1 \\ 4 & 1 & 6 \end{bmatrix} \quad (6)$$

Displacement Field

The displacement undergone by any point on a body can be expressed as a function of original coordinates. The displacement field U is expressed as

$U = ui + vj + wk$

This function is known as displacement field vector where,

$u = f_1(x,y,z)$

$v = f_2(x,y,z)$

$w = f_3(x,y,z)$

STRAIN COMPONENTS (STRAIN – DISPLACEMENT RELATIONS)

Strain Component along x direction,

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]$$

Strain Component along y direction,

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

Strain Component along z direction,

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

STRAIN COMPONENTS (STRAIN – DISPLACEMENT RELATIONS)

Shear Strain in the xy plane,

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

Shear Strain in the yz plane,

$$\gamma_{yz} = 2\varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial z}$$

Shear Strain in the xz plane,

$$\gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial z}$$

STRAIN DISPLACEMENT RELATIONS – (LINEAR TERMS ONLY)

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = 2\varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \qquad \gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

STATE OF STRAIN AND STRAIN TENSOR AT A POINT

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_{zz} \end{bmatrix}$$

State of Strain at a point

$$\begin{bmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{bmatrix}$$

Strain Tensor

ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 1

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

STRESS TENSOR

$$\begin{bmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_{zz} \end{bmatrix}$$

STRAIN TENSOR

$$\tau_{xy} = \tau_{yx} \; ; \; \tau_{xz} = \tau_{zx} \; ; \; \tau_{zx} = \tau_{zy}$$

$$\gamma_{xy} = \gamma_{yx} \; ; \; \gamma_{yz} = \gamma_{zy} \; ; \; \gamma_{xz} = \gamma_{zx}$$

ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 2

$$\varepsilon_{PQ} = \varepsilon_{xx} n_x^2 + \varepsilon_{yy} n_y^2 + \varepsilon_{zz} n_z^2 +$$

$$2 \text{ x } \left(\frac{1}{2}\gamma_{xy} n_x n_y\right) + 2 \text{ x } \left(\frac{1}{2}\gamma_{yz} n_y n_z\right) + 2 \text{ x } \left(\frac{1}{2}\gamma_{xz} n_x n_z\right)$$

This is analogous to the stress relation:

$$\sigma_n = n_x T_x^n + n_y T_y^n + n_z T_z^n$$

$$\sigma^n = \sigma_{xx} n_x^2 + \tau_{xy} n_x n_y + \tau_{xz} n_x n_z + \sigma_{yy} n_y^2 + \tau_{yx} n_y n_x + \tau_{yz} n_y n_z + \sigma_{zz} n_z^2 + \tau_{zx} n_z n_x + \tau_{zy} n_z n_y$$

$$\sigma^n = \sigma_{xx} n_x^2 + \sigma_{yy} n_y^2 + \sigma_{zz} n_z^2 + 2\tau_{xy} n_x n_y + 2\tau_{yz} n_y n_z + 2\,\tau_{xz} n_x n_z$$

STRAIN TRANSFORMATION EQUATION

The above Transformation equations can be

written in the Matrix Form as:

$$[\varepsilon]_{x'y'z'} = [\alpha]^T[\varepsilon][\alpha]$$

This is analogous to

$$[\sigma]_{x'y'z'} = [\alpha]^T[\sigma][\alpha]$$

$$[\varepsilon]_{x'y'z'} = \begin{bmatrix} \varepsilon_{x'x'} & \frac{1}{2}\gamma_{x'y'} & \frac{1}{2}\gamma_{x'z'} \\ \frac{1}{2}\gamma_{x'y'} & \varepsilon_{y'y'} & \frac{1}{2}\gamma_{y'z'} \\ \frac{1}{2}\gamma_{x'z'} & \frac{1}{2}\gamma_{y'z'} & \varepsilon_{z'z'} \end{bmatrix}$$

$$[\varepsilon]_{xyz} = \begin{bmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{bmatrix}$$

$$[\alpha] = \begin{bmatrix} n_{xx'} & n_{xy'} & n_{xz'} \\ n_{yx'} & n_{yy'} & n_{yz'} \\ n_{zx'} & n_{zy'} & n_{zz'} \end{bmatrix}$$

ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 4

PRINCIPAL STRAIN

$$\begin{vmatrix} \varepsilon_{xx}-\varepsilon & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy}-\varepsilon & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz}-\varepsilon \end{vmatrix} = 0$$

$$\varepsilon^3 - J_1\varepsilon^2 + J_2\varepsilon - J_3 = 0$$

$$J_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$J_3 = \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{vmatrix}$$

$$J_2 = \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} \end{vmatrix} + \begin{vmatrix} \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{vmatrix} + \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xz} & \varepsilon_{zz} \end{vmatrix}$$

PRINCIPAL STRAIN

$$(\varepsilon_{xx} - \varepsilon)n_x + \frac{1}{2}\gamma_{xy}n_y + \frac{1}{2}\gamma_{xz}n_z = 0$$

$$\frac{1}{2}\gamma_{yx}n_x + (\varepsilon_{yy} - \varepsilon)n_y + \frac{1}{2}\gamma_{yz}n_z = 0$$

$$\frac{1}{2}\gamma_{zx}n_x + \frac{1}{2}\gamma_{zy}n_y + (\varepsilon_{zz} - \varepsilon)n_z = 0$$

$$n_x^2 + n_y^2 + n_z^2 = 1$$

NOTE-1

Stress invariants in terms of Principal Stresses:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_1 \sigma_3$$

$$I_3 = \sigma_1 \sigma_2 \sigma_3$$

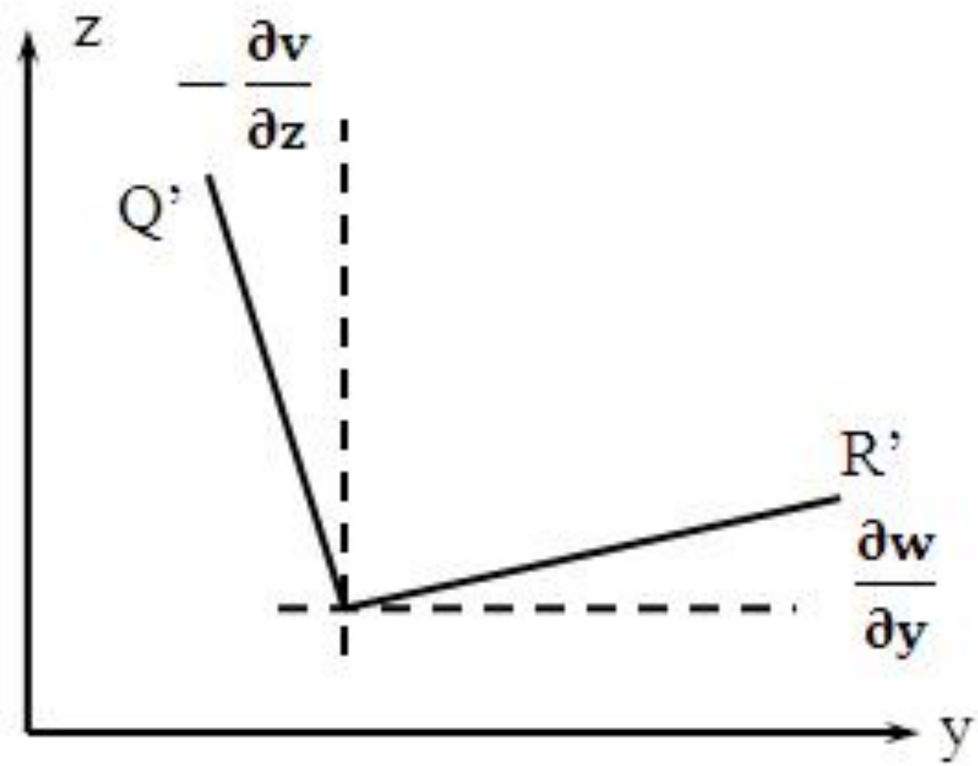
Strain invariants in terms of Principal Strains:

$$J_1 = \epsilon_1 + \epsilon_2 + \epsilon_3$$

$$J_2 = \epsilon_1 \epsilon_2 + \epsilon_2 \epsilon_3 + \epsilon_1 \epsilon_3$$

$$J_3 = \epsilon_1 \epsilon_2 \epsilon_3$$

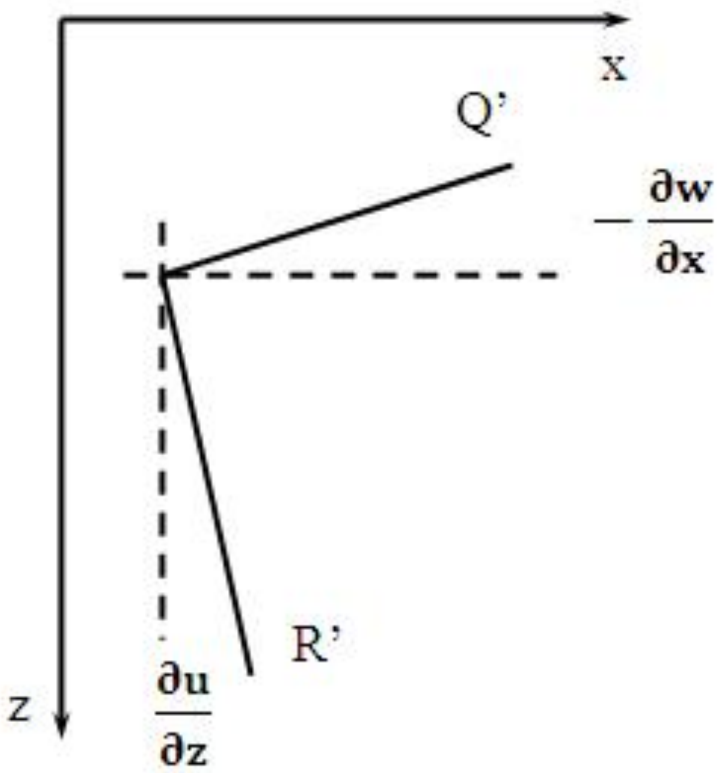
RIGID BODY ROTATIONS



Rotation about the x axis:

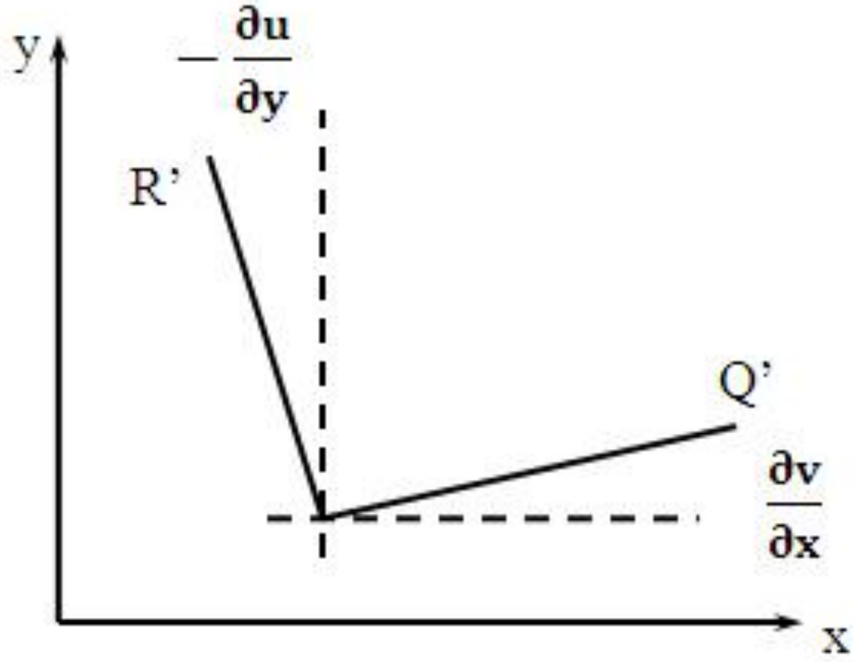
$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

RIGID BODY ROTATIONS



Rotation about the y axis: $$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

RIGID BODY ROTATIONS



Rotation about the z axis: $$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

MAX SHEAR STRESS & STRAIN

Maximum Shear Stress: $$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Maximum Shear Strain: $$\tfrac{1}{2}\gamma_{max} = \frac{\varepsilon_{max} - \varepsilon_{min}}{2}$$

Principal Strain for 2D state of strain

$$\varepsilon_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \left(\tfrac{1}{2}\gamma_{xy}\right)^2}$$

$$\varepsilon_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \left(\tfrac{1}{2}\gamma_{xy}\right)^2}$$

$$\text{Tan } 2\theta = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$$

The state of strain at a point is given by

$$[\varepsilon_{ij} = \begin{bmatrix} 0.02 & -0.04 & 0 \\ -0.04 & 0.06 & -0.02 \\ 0 & -0.02 & 0 \end{bmatrix}$$

In the direction PQ having cosines nx = 0.6; ny = 0 and nz = 0.8; Determine ε_{PQ}

The displacement field for a body is given below

$$\mathbf{U} = \left(\mathbf{x}^2 + \mathbf{y}\right)\mathbf{i} + (\mathbf{3} + \mathbf{z})\mathbf{j} + \left(\mathbf{x}^2 + \mathbf{2y}\right)\mathbf{k}$$

Determine the principal strains at (3,1,-2) and the direction of minimum strain. Use only linear terms

COMPATIBILITY CONDITIONS

COMPATIBILITY CONDITIONS

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z}$$

COMPATIBILITY CONDITIONS

The three displacement components and six strain components are related by six strain displacement relations of Cauchy.

The determination of six strain components from three displacement components involves only differentiation.

However the reverse operation that is determination of three displacement components from six strain components is more complicated. Since it involves integrating six equations to obtain 3 functions.

COMPATIBILITY CONDITIONS

Therefore all strain components cannot be prescribed arbitrarily and there must exist a definite relation among the strain components. This relation among strain components is called **compatibility equations**

COMPATIBILITY CONDITIONS

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial v}{\partial x}\right) \Rightarrow \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z}$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z}$$

COMPATIBILITY CONDITIONS

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\frac{\partial \gamma_{xy}}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 v}{\partial x \partial z}$$

$$\frac{\partial \gamma_{yz}}{\partial x} = \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y}$$

$$\frac{\partial \gamma_{xz}}{\partial y} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 w}{\partial x \partial y}$$

COMPATIBILITY CONDITIONS

$$\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} = 2\frac{\partial^2 w}{\partial x \partial y}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right) = 2\frac{\partial^3 w}{\partial x \partial y \partial z}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right) = 2\frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y}\right) = 2\frac{\partial^2 \varepsilon_{yy}}{\partial x \partial z}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x}\right) = 2\frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z}$$

COMPATIBILITY CONDITIONS

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$$\frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z}$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z}$$

I

$$\frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right) = 2\frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y}\right) = 2\frac{\partial^2 \varepsilon_{yy}}{\partial x \partial z}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x}\right) = 2\frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z}$$

II

COMPATIBILITY CONDITIONS

State the conditions under which the following is a possible system of strains:

$$\varepsilon_{xx} = a + b(x^2 + y^2)\,x^4 + y^4, \qquad \gamma_{yz} = 0$$
$$\varepsilon_{yy} = \alpha + \beta(x^2 + y^2) + x^4 + y^4, \qquad \gamma_{zx} = 0$$
$$\gamma_{xy} = A + Bxy\,(x^2 + y^2 - c^2), \qquad \varepsilon_{zz} = 0$$

COMPATIBILITY CONDITIONS

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[*Ans.* $B = 4;\ b + \beta + 2c^2 = 0$]